Measuring Sparticle Masses Using Transverse Mass Kink

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Based on
W.S. Cho, KC, Y.G. Kim & C.B. Park,
arXiv : 0709.0288 [hep-ph], 0711.4526 [hep-ph]

More details and collider applications will be discussed in Y.G. Kim's talk.

◆ New Physics at TeV ?

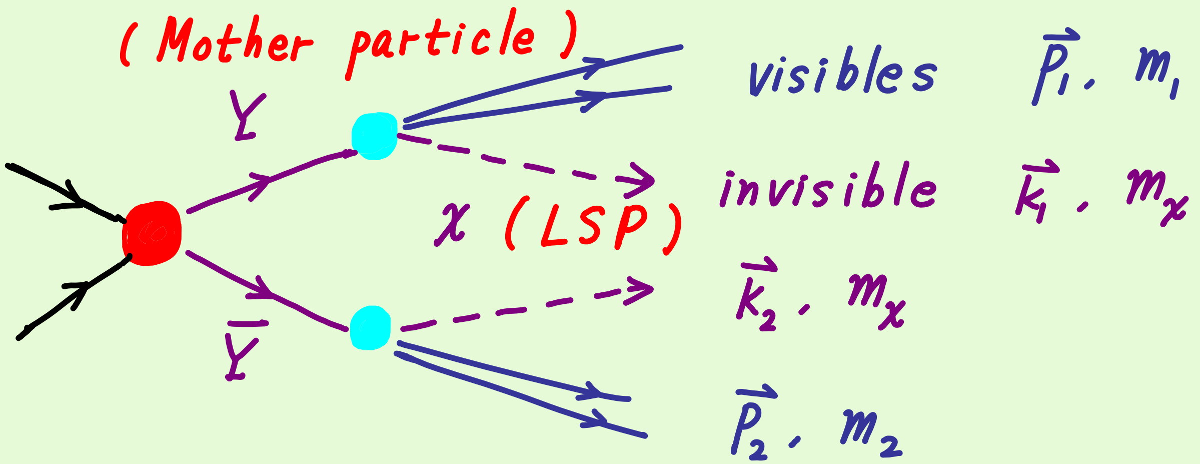
- Hierarchy problem
- Dark matter : WIMP ?
- Constraints from precision EW measurements, p-decay, etc

⇒ New particles at TeV with conserved quantum number (SUSY with R-parity, Little Higgs with T-parity, UED with KK-parity, …)

- Typical Collider Signal

Pair-produced new particles $Y + \bar{Y}$ decaying as

$Y \to$ visible particle(s) + an invisible particle χ



$\vec{p}_i$ = Total visible momentum from Y ($\bar{Y}$)

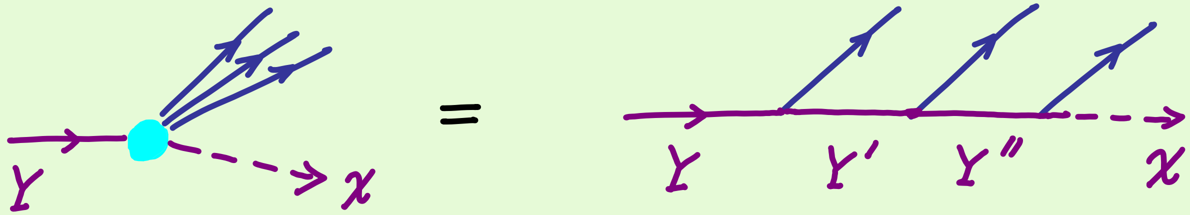
m_i = Total invariant mass of $\vec{p}_i$

Unknowns : m_Y , m_χ , $\vec{k}_i$

- Determination of m_Y, m_χ, $\cdots$

Long decay chain

Hinchliffe et al. (1997);
Allanach et al. (2000);
Weiglein et al. (2006)



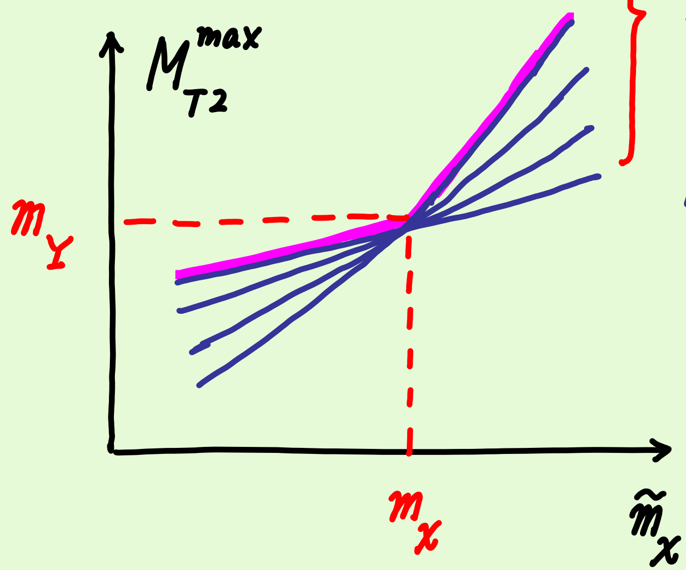
Many invariant mass combinations of visible particles whose end point values & distribution shapes provide information on the involved new particle masses.

In many cases, the overall mass scale is uncertain, while the mass differences are accurately determined.

Also the new particle mass spectra might not allow such a long cascade decay.

Transverse mass (M_{T2}) kink

Cho, KC, Kim, Park (2007)



Introduce a trial LSP mass $\tilde{m}_\chi$, and extrapolate m_Y to the hypothetical situation with $\tilde{m}_\chi \neq m_\chi$, giving different curves for different visible kinematic variables.

$\tilde{m}_\chi$ = trial LSP mass

The global maximum of M_{T2} shows a kink structure at $\tilde{m}_\chi = m_\chi$: simultaneous determination of m_χ & m_Y.

The shape of $M_{T2}^{max}(\tilde{m}_\chi)$ at $\tilde{m}_\chi > m_\chi$ can provide further information on intermediate on-shell particle mass.

♦ M_{T2} : Generalized Transverse Mass

Lester, Summers (1999) ; Barr, Lester, Stephens (2003)

- Transverse mass : M_T

$W \to \ell(p)\ \nu(k)$ ← missing momentum, but k_T can be determined for each event

$$p^\mu = (E_T \cosh\eta ,\ \vec{p}_T ,\ E_T \sinh\eta)$$

$$\eta = \tfrac{1}{2} \ln \left(E+p_z / E-p_z \right) , \quad E_T(\vec{p}_T) = \sqrt{|\vec{p}_T|^2 + m^2}$$

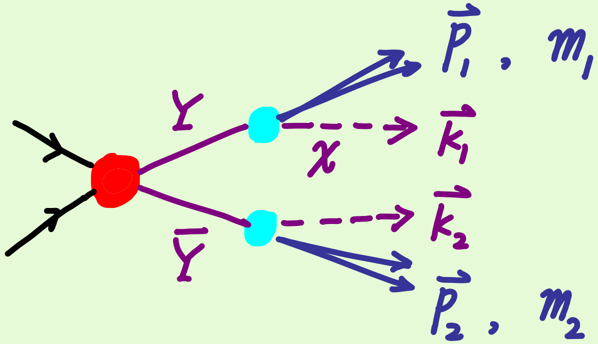
$$M_T^2 = m_\ell^2 + m_\nu^2 + 2 \left(E_T(\vec{p}_T) E_T(\vec{k}_T) - \vec{p}_T \cdot \vec{k}_T \right)$$

$$M_W^2 = m_\ell^2 + m_\nu^2 + 2 \left(E_T(\vec{p}_T) E_T(\vec{k}_T) \cosh\Delta\eta - \vec{p}_T \cdot \vec{k}_T \right) \geq M_T^2$$

$$\Rightarrow \quad M_T^{max} = \underset{\text{all data}}{\text{Maximum over}} \left[M_T (\underbrace{\vec{p}_T , \vec{k}_T , m_\ell , m_\nu}_{\text{all measured or known}}) \right] = M_W$$

- M_{T2} for events with two missing particles

Lester, Summers (1999)



$\vec{k}_1$, $\vec{k}_2$, m_χ are unknown except for $\vec{k}_{1T} + \vec{k}_{2T}$

$$\left(M_T^{(i)}\right)^2 = m_i^2 + \tilde{m}_\chi^2 + 2\left(\sqrt{|\vec{P}_{iT}|^2 + m_i^2}\,\sqrt{|\vec{k}_{iT}|^2 + \tilde{m}_\chi^2} - \vec{P}_{iT}\cdot\vec{k}_{iT}\right)$$

$(i = 1, 2)$ trial LSP mass trial LSP momenta

$$M_{T2}(\vec{P}_{iT}, m_i, \tilde{m}_\chi) = \min_{\{\vec{k}_{iT}\}} \left[\max\left(M_T^{(1)}, M_T^{(2)}\right) \right]$$

$\hookrightarrow \{\vec{k}_{1T}, \vec{k}_{2T} : \vec{k}_{1T} + \vec{k}_{2T} = -\vec{P}_{1T} - \vec{P}_{2T}\}$

$$M_{T2}^{max}(\tilde{m}_\chi) = \max_{\{\vec{P}_{iT},\, m_i\}} \left[M_{T2}(\vec{P}_{iT}, m_i, \tilde{m}_\chi) \right]$$

= Hypothetical m_Y for hypothetical LSP mass $\tilde{m}_\chi$

$$M_{T2}(\vec{P}_{iT}, m_i, \tilde{m}_\chi) = \min_{\{\vec{k}_{iT}\}} \left[\max\left(M_T^{(1)}, M_T^{(2)} \right) \right]$$

$T \equiv$ momentum space along which $\vec{P}_Y + \vec{P}_{\bar{Y}}$ is known.

$$\left(\dim(T) = \begin{cases} 2 & \text{for hadron collider} \\ 3 & \text{for lepton collider} \end{cases} \right)$$

Analytic form of M_{T2} for $\dim(T) = 2$:

Lester & Barr (2007)

$$M_{T2}^2 = \begin{cases} \text{Unbalanced solution :} \quad (m_i + \tilde{m}_\chi)^2 \\ \text{or} \\ \text{Balanced solution :} \quad \tilde{m}_\chi^2 + A \\ \qquad + \left(\left(1 + \dfrac{4\tilde{m}_\chi^2}{2A - m_1^2 - m_2^2} \right) \left(A^2 - m_1^2 m_2^2 \right) \right)^{1/2} \\ \qquad (A = E_{1T} E_{2T} + \vec{P}_{1T} \cdot \vec{P}_{2T}) \end{cases}$$

M_{T2} is invariant under the spacial rotation & back-to-back Lorentz boost of $\vec{P}_{1T}$ & $\vec{P}_{2T}$.

Cho, KC, Kim, Park (2007)

Combined with the definition of M_{T2}, this implies M_{T2} is the same function of the 4 invariant variables m_1, m_2, $A = E_{1T}E_{2T} + \vec{P}_{1T}\cdot\vec{P}_{2T}$, $\tilde{m}_\chi$ for dim(T) = 1, 2, 3, ⋯

It is straightforward to derive M_{T2} for dim(T) = 1,

$$\Rightarrow \quad M_{T2}^2 = \begin{cases} (m_i + \tilde{m}_\chi)^2 \\ \quad \text{or} \\ \tilde{m}_\chi^2 + A + \left(\left(1 + \dfrac{4\tilde{m}_\chi^2}{2A - m_1^2 - m_2^2} \right) \left(A^2 - m_1^2 m_2^2 \right) \right)^{1/2} \end{cases}$$

Maximizing M_{T2} over all events

$$M_{T2}^{max}(\tilde{m}_\chi) = \max_{\{\vec{p}_{iT},\, m_i\}} \left[M_{T2}(\vec{p}_{iT}, m_i ; \tilde{m}_\chi) \right]$$

Again, using the back-to-back boost invariance, one can show that the global maximum can be obtained by considering $Y + \bar{Y}$ at rest :

$$M_{T2}^{max}(\tilde{m}_\chi) = \max_{\{m_1, m_2, \theta\}} \Big[\underbrace{\mathcal{F}(m_1, m_2, \theta, \tilde{m}_\chi)}_{M_{T2} \text{ for } Y+\bar{Y} \text{ at rest}} \Big]$$

$$\mathcal{F}(m_1, m_2, \theta, \tilde{m}_\chi) = M_{T2}(m_1, m_2, A : \tilde{m}_\chi)$$

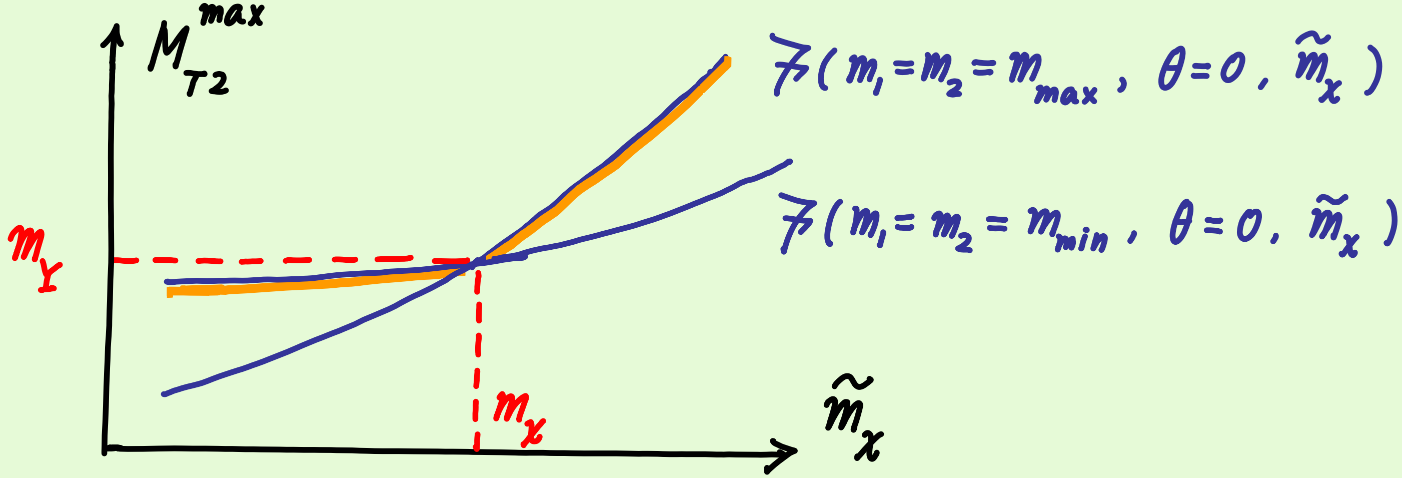
$$\text{for} \quad A = \sqrt{p_{1T}^2 + m_1^2}\sqrt{p_{2T}^2 + m_2^2} + p_{1T}p_{2T}\cos\theta$$

$$p_{iT}^2 = \left((m_Y + m_i)^2 - m_\chi^2\right)\left((m_Y - m_i)^2 - m_\chi^2\right) / 4m_Y^2$$

It is straightforward to find

$$M_{T2}^{max}(\tilde{m}_\chi) = \begin{cases} \mathcal{F}(m_1 = m_2 = \overset{\text{maximal visible invariant mass}}{m_{max}},\ \theta = 0,\ \tilde{m}_\chi) & \text{for } \tilde{m}_\chi > m_\chi \\ \mathcal{F}(m_1 = m_2 = \overset{\text{minimal visible invariant mass}}{m_{min}},\ \theta = 0,\ \tilde{m}_\chi) & \text{for } \tilde{m}_\chi < m_\chi \end{cases}$$

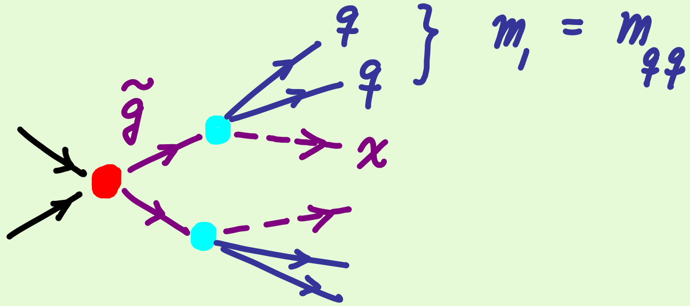
$$\mathcal{F}(\underbrace{m_1 = m_2}_{m_v},\ \theta = 0,\ \tilde{m}_\chi) = \frac{1}{2m_\Upsilon}\left[m_\Upsilon^2 - m_\chi^2 + m_v^2 + \sqrt{(m_\Upsilon^2 + m_\chi^2 - m_v^2)^2 + 4m_\Upsilon^2(\tilde{m}_\chi^2 - m_\chi^2)}\,\right]$$



Kink structure appears generically if the decay products of Υ contains more than one visible particles, so that $m_{min} < m_{max}$

• Example : Gluino M_{T2}

$$\tilde{g}\tilde{g} \to q q \chi \; q q \chi$$

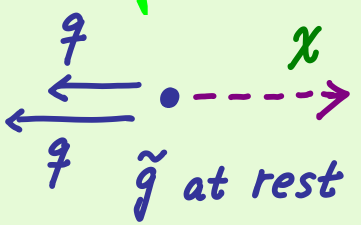


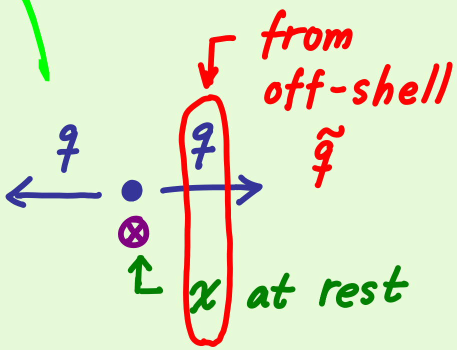
Three body decay

$$\tilde{g} \to q q \chi$$

$$(m_{\tilde{q}} > m_{\tilde{g}})$$

$$0 \le m_{qq} \le m_{\tilde{g}} - m_{\chi}$$



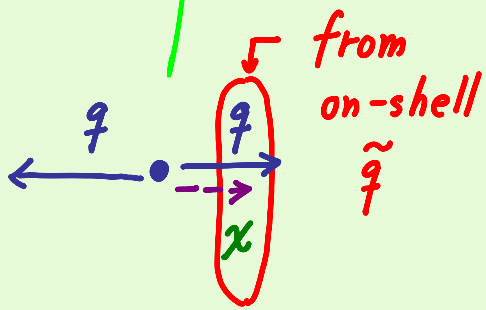


Cascade two body decays

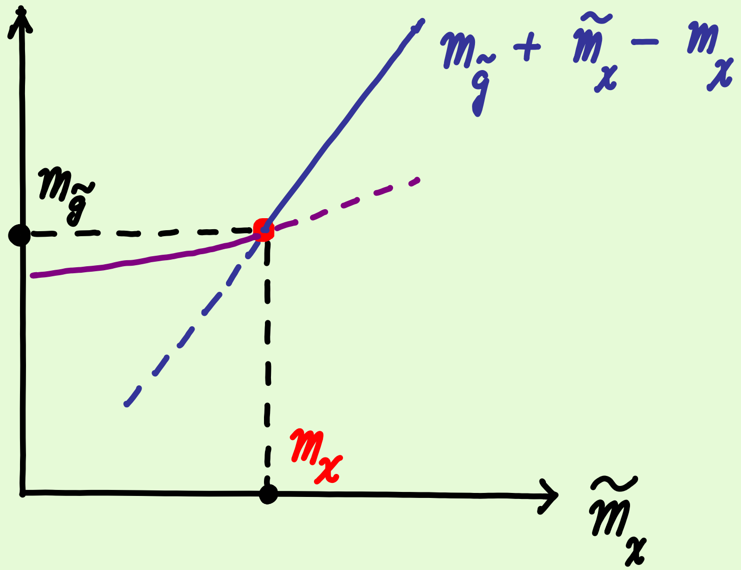
$$\tilde{g} \to \tilde{q} q \to q q \chi$$

$$(m_{\tilde{q}} < m_{\tilde{g}})$$

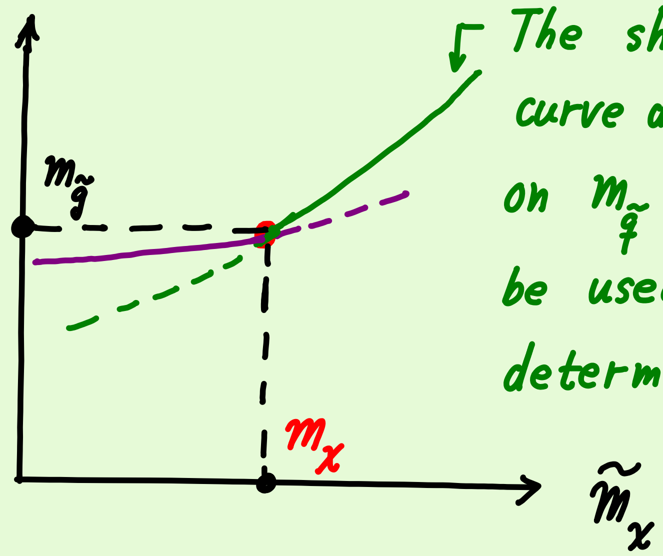
$$0 \le m_{qq} \le \frac{\sqrt{(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)(m_{\tilde{q}}^2 - m_{\chi}^2)}}{m_{\tilde{q}}}$$



Three body decay



Cascade two body decays



The shape of curve depends on $m_{\tilde{q}}$, so can be used to determine $m_{\tilde{q}}$.

Kink structure is sharper when $m_{\tilde{g}} - m_{\tilde{q}}$ is more sizable.

Transverse mass kink method can be applied to a variety of mother particle pairs & the resulting visible particle combinations.

Summary

- Transverse mass (M_{T2}) kink might provide a useful way to determine the overall mass scale of new particles at TeV, particularly the WIMP dark matter mass.

- It might be possible to consider a variant of M_{T2} to make the kink structure sharper.
  (Work in progress)

- More details and applications to LHC will be discussed in Y.G. Kim's talk.